A Study on Crop Planting Strategies Based on Integer Linear Programming Models and Monte Carlo Simulation Algorithms

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Abstract: This paper presents a comprehensive strategy for optimizing crop planting decisions based on Integer Linear Programming (ILP) models and Monte Carlo Simulation (MCS) algorithms. The study begins with data preprocessing, which involves calculating the average selling prices of various crops and determining the expected sales volumes using characteristic equations. Considering land-type constraints, crop rotation requirements, and planting dispersion constraints, an ILP model is formulated to maximize farming profits over the period from 2024 to 2030. To address the uncertainties associated with input parameters, a dynamic stochastic agricultural planning model is developed. This model employs state variables to capture the dynamic changes in prices, yields, costs, and market demands. By integrating dynamic programming with MCS, multiple iterations are conducted to handle these uncertainties. The results are visualized through heatmaps, depicting the dynamic trends of planting acreage and profits for different crops.

1. Introduction

Agricultural planning is a complex task that requires precise management of resources to maximize profits while adhering to various constraints. Traditional approaches often overlook the dynamic nature of market demands, weather conditions, and crop characteristics, leading to suboptimal planting strategies. In this paper, we propose a sophisticated methodology that leverages advanced mathematical programming and simulation techniques to address these challenges.

The research conducted by Mahmood Fazlali and colleagues [1] explored the challenge of load balancing in parallel branch-and-bound (B&B) algorithms, specifically focusing on subtree parallelism. These algorithms exhibited effectiveness in resolving MILP models stemming from behavioral synthesis. In another study, Ameni Kraiem et al. [2] formulated a mixed-integer linear programming model aimed at minimizing overall travel time and applied it to an instance of Cordeau's multi-vehicle lot vehicle routing problem to validate a newly created scenario. Gustavo A. Cardona et al. [3] introduced an efficient control synthesis based on optimization for Signal Timing Logic (STL) and its extension, known as weighted Signal Timing Logic (wSTL). While STL encompasses Boolean and temporal operators, wSTL further empowers users to articulate preferences and priorities for concurrent and sequential tasks through the weighting of logical and temporal operators, along with fulfilling time representations. Shiyuan Yang and team [4] presented an enhanced generalized augmented simulation (ES) scaling formulation, which integrates soft Monte Carlo simulation with a generalized ES scaling formulation and an SVR model to assess failure probability. Mohammad Firouz and colleagues [5], utilizing a sliced tree representation, a hybrid genetic algorithm, and a simulation-based evaluation framework, devised a method that efficiently balances material handling efficiency and rescheduling costs, ensuring resilient layouts in stochastic scenarios.

The first step in our approach involves rigorous data preprocessing. By calculating the average selling prices of crops and utilizing characteristic equations to estimate expected sales volumes, we establish a solid foundation for subsequent modeling. Next, we formulate an Integer Linear Programming (ILP) model, which is well-suited for optimization problems involving discrete decision variables and linear constraints. In this context, the ILP model incorporates multiple

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constraints, such as land-type suitability, crop rotation requirements, and planting dispersion, to ensure that the resulting planting strategy is both feasible and profitable.

However, agricultural planning is inherently uncertain due to fluctuations in market prices, weather patterns, and crop yields. To address these uncertainties, we develop a dynamic stochastic agricultural planning model. This model captures the dynamic changes in key variables using state space representation and employs a combination of dynamic programming and Monte Carlo Simulation (MCS) to handle the inherent stochasticity. By iteratively simulating various scenarios, we can assess the robustness of the proposed planting strategy and identify potential risks and opportunities.

2. Modeling and solving integer linear programming models

2.1 Decision variables

Specify whether the jth crop is grown on the i-th plot as n_{tijk}, which is a binary variable where t denotes the year in which it is located and k denotes the quarter, as specified:

$$CO_{it} = \alpha_1 + \beta_1 \mathrm{did}_{it} + \gamma_1 X_{it} + \mu_i + \mu_t + \varepsilon_{it} \tag{1}$$

(a) Crop area.

The area of the jth crop planted in the i-th plot in year t is X_{tijk}, where $(t = 2024,2025,\cdots,2030; i = 1,2,\cdots,54; j = 1,2,\cdots,41; k = 1,2)$, with $k = 1$ indicating the first season; $k = 2$ indicating the second season.

At the same time, it is stipulated that only one season of crops can be planted per year, and when $X_{tij1} = 0$, X_{tij2} can be 1 or 0, and similarly, when $X_{tij2} = 0$, X_{tij2} can be 1 or 0.

(b) Total crop production:

The total production of the jth crop in the i-th plot in year t is denoted as Q_{tij}.

$$Q_{tij} = \sum_{t=2024}^{2030} \sum_{k=1}^{2} \sum_{i=1}^{54} n_{tijk} X_{tijk} Y_j (j = 1,2,\cdots 41) \tag{2}$$

where n_{tijk} indicates whether the jth crop is grown on the ith plot and Yj is the acre yield of the jth crop (41 crops in total).

(c) Expected sales volume:

Assuming that the production of the jth crop is its sales volume, the expected sales volume of the jth crop is denoted as E_j .

2.2 Objective function

Maximize the profit earned in 2024-2030:

$$\mathrm{Max}W = \sum_{t=2024}^{2030} \sum_{k=1}^{2} \sum_{i=1}^{54} \sum_{j=1}^{41} \left(Q_{tj} P_j - n_{tijk} X_{tijk} C_j \right) \tag{3}$$

where W represents the profit earned from 2024 to 2030.

If the crop production does not exceed the expected sales volume, the crop can be marketed normally and the excess will not be marketed, which can be obtained as:

$$W = \sum_{t=2024}^{2030} \sum_{k=1}^{2} \sum_{i=1}^{54} \sum_{j=1}^{41} \left(min\left(E_j, Q_{tij}\right) \cdot P_j - nX_{tij} C_j \right) \tag{4}$$

If the crop production does not exceed the expected sales volume, then the crop can be sold normally, yielding:

$$W = \sum_{t=2024}^{2030} \sum_{k=1}^{2} \sum_{i=1}^{54} \sum_{j=1}^{41} \left(Q_{tij}P_j - X_{tij}C_j\right) \tag{5}$$

The excess is sold at a 50% reduction in price.

$$W = \sum_{t=2024}^{2030} \sum_{k=1}^{2} \sum_{i=1}^{54} \sum_{j=1}^{41} \left(Q_{tij}P_j - X_{tij}C_j\right) \tag{6}$$

2.3 Constraints

1) The area of each plot planted with crops cannot exceed the total area:

$$\sum_{j=1}^{54} X_{tijk} \leq A_i \tag{7}$$

2) Plant beans in each plot at least once every three years:

$$\sum_{i=1}^{3} T_{ij} \geq 1 \tag{8}$$

where T_{ij} is whether or not each plot is planted with beans:

$$T_{ij} = \begin{cases} 1 \text{ Growing beans} \\ 0 \text{ No beans were planted} \end{cases} \tag{9}$$

3) Only suitable crops can be grown on different plots:

Flat dry land, terraced land and hillside land can only grow one crop per year. Watered land can grow one or two crops per year. Greenhouses provide a degree of insulation and can grow two seasons of crops per year. Smart greenhouses mainly use solar energy to automatically adjust the temperature in the shed in winter to ensure the normal growth of crops.

4) The same plot of land (including greenhouses) cannot be planted with the same crop:

$$X_{tijk} \cdot X_{(t+1)jk} = 0 \tag{10}$$

5) In order to facilitate field management, the planting area of each crop should not be too small, and it may be useful to specify the minimum area, denoted as Squaremin. Where $\text{Square}_{min} = 0.25A_i$

$$X_{tijk} \geq \text{Square}_{min} \tag{11}$$

6) Since each crop cannot be spread too thinly, the total number of plantings of the same crop in different plots cannot exceed $N_{j,\max}$, based on the number of times the jth crop is planted in 2023, and then the following relationship can be obtained:

$$\sum_{i=1}^{54} n_{tijk} \leq N_{j,max} \tag{12}$$

7) Non-negative constraints:

From the actual situation, the area X_{tijk} of the i-th plot planted with the j-th crop in year t is shown to be non-negative:

$$X_{tijk} \geq 0 \tag{13}$$

8) Constraints on ordinary greenhouses.

Ordinary greenhouses grow two seasons of crops per year; in the first season, a wide range of vegetables can be grown (with the exception of cabbage, white radish and carrot), and in the second season, only edible mushrooms can be grown.

$$\sum_{j\in \text{vegetable}} n_{tij1} \leq 1. \forall i \in (\text{General shed}) \tag{14}$$

$$\sum_{j\in \text{edible mushroom}} n_{tij2} \leq 1. \forall i \in (\text{General shed}) \tag{15}$$

9) Constraints on smart greenhouses.

Smart greenhouses are allowed to grow two seasons of vegetables every year (except cabbage, white radish and carrot):

$$\sum_{j\in \text{vegetable}} \sum_{k=1}^{2} n_{ijj} \leq 2, \forall i \in (\text{Smart Shed}) \tag{16}$$

2.4 Model Solving

In constructing the integer planning model, we delve into the complex relationships between decision variables, objective functions, and constraints, while applying optimization algorithms for systematic analysis. We utilize the Python programming language to implement and solve the established multidimensional agricultural planning optimization model in order to determine the mix of crops to be planted in each plot in order to maximize the overall yield. Given that two different cropping strategies (strategy (1) and strategy (2)) are explicitly presented in the title, this study will discuss the optimization model in a careful categorization to ensure that the level of maximized returns that can be achieved by each strategy can be comprehensively and accurately assessed under different strategy requirements.

First, strategy (1) may focus on the cultivation of a specific crop, possibly based on market demand, cost-benefit analysis, or specific environmental conditions. Strategy (2), on the other hand, may consider a wider range of factors, such as long-term ecological balance, diversification of planting to counteract risks, or the use of specific technologies to improve production efficiency. By classifying the model into these two strategy categories, we are able to more precisely analyze and compare the optimal solutions under different strategies, thus providing a more comprehensive basis for decision makers.

When discussing the categorization, we will first identify the key variables and constraints under each strategy, including but not limited to crop type, planting area, expected yield, cost inputs, market selling price, environmental impacts, and other factors.

2.5 Analysis and visualization of results

Analysis of the results leads to the following important findings:

1) Crop rotation: Checking the planting of legumes to confirm that the requirement of planting at least once every three years is being met.

This is essential for maintaining soil fertility and achieving long-term sustainability.

2) Land use patterns: Look at how different types of land are cultivated to see how the model utilizes different land characteristics to maximize returns.

Land use patterns: Look at how different types of land are cultivated to see how the model uses the characteristics of different lands to maximize returns. For example, it was found that irrigated land was used more for growing high-value vegetables than rice.

3) Cropping structure: Observe the distribution of acreage planted with different crops. Some high-value or high-demand crops occupy larger acreage, while low-yield crops are planted only

minimally to meet crop rotation requirements, which reflects the model's trade-off between economic efficiency and sustainability.

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4) Seasonal cropping patterns: Seasonal cropping patterns can be observed when comparing cropping across seasons, reflecting the growth cycles of different crops and the fact that they are grown in different seasons.

Seasonal cropping patterns can be observed when comparing cropping in different seasons, reflecting seasonal variations in the growth cycles of different crops and in market demand.

5) Impact of overcropping: Compare the results of two overcropping scenarios and analyze the impact of the overcropping treatment on planting decisions.

Impact of over-yielding: Compare the results of the two over-yielding treatments and analyze the impact of the over-yielding treatment on planting decisions. For example, an increase in the area planted with certain high-yielding crops is seen in the case where the excess yield can be sold at 50%.

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In order to show these results more directly, we perform further data visualization, which allows us to fully understand the planting decisions given by the optimization model.

This validates the model and provides farmers and policy makers with an intuitive and understandable way to make decisions.

It provides farmers and decision makers with intuitive and easy-to-understand decision support. For example, it may be found that the area planted to certain high-value crops can be increased. At the same time the optimal planting strategies are different in the two over-yield treatment scenarios, which can provide targeted advice to farmers and help them to make better planting decisions according to the actual market situation.

Part of the solution results are shown in Figure 1.

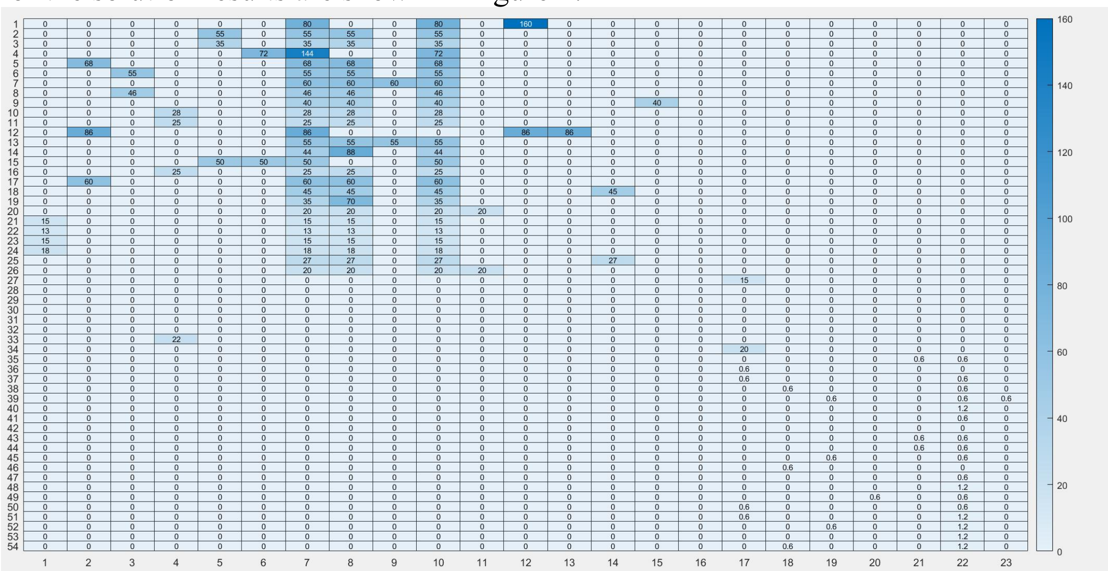

Fig. 1 Heat map of planting trends.

3. Modeling with uncertainty considerations

Due to the introduction of uncertain factors such as future sales, planting costs and mu yield, these factors have a certain degree of volatility and uncertainty. Therefore, it is necessary to adjust the model on the basis of Problem 1, and then solve the optimal planting program after the introduction of these uncertain factors.

1) Uncertain factors

Changes in mu yield: mu yield will be affected by climate and other factors, there are changes of ± 10% per year.

Changes in expected sales volume: wheat and corn have a growing trend in the future, with an average annual growth rate of 5%-10%.

The expected future sales volume of other crops is expected to change by ±5% per year relative to 2023.

Changes in growing costs: Growing costs are expected to increase by about 5% per year on average.

Fluctuation of sales price: grain crops are basically stable, vegetable crops have a growing trend, with an average growth rate of about 5% per year; the sales price of edible fungus is stable and decreasing, with a decrease of about 1%-50% per year, especially the sales price of morel mushrooms with a decrease of 5% per year.

2) Uncertain parameters

Expected rate of change of sales volume δ: The expected rate of change of sales volume of wheat is δ_1, with a range from 0.05 to 0.10, and the expected rate of change of sales volume of other crops is δ_2, with a range from -0.05 to 0.05.

Rate of change of yield per acre ζ_j : The rate of change of yield per acre of the jth crop is noted as ζ_j can be known to range from -0.1 to 0.1.

Rate of increase in cost of cultivation of crops η : Where $\eta = 0.05$

Rate of change of selling price ξ : Remember the rate of change of selling price of vegetables as ξ_1, $\xi_1 = 0.05$; remember the rate of change of selling price of edible mushrooms as ξ_2, where ξ_2 ranges from -0.01 to 0.05; and remember the rate of change of selling price of morel mushrooms as ξ_3, where $\xi_3 = 0.05$

Establishing a mathematical model similar to the one in Section 2 the results can be obtained as shown in Figure 2.

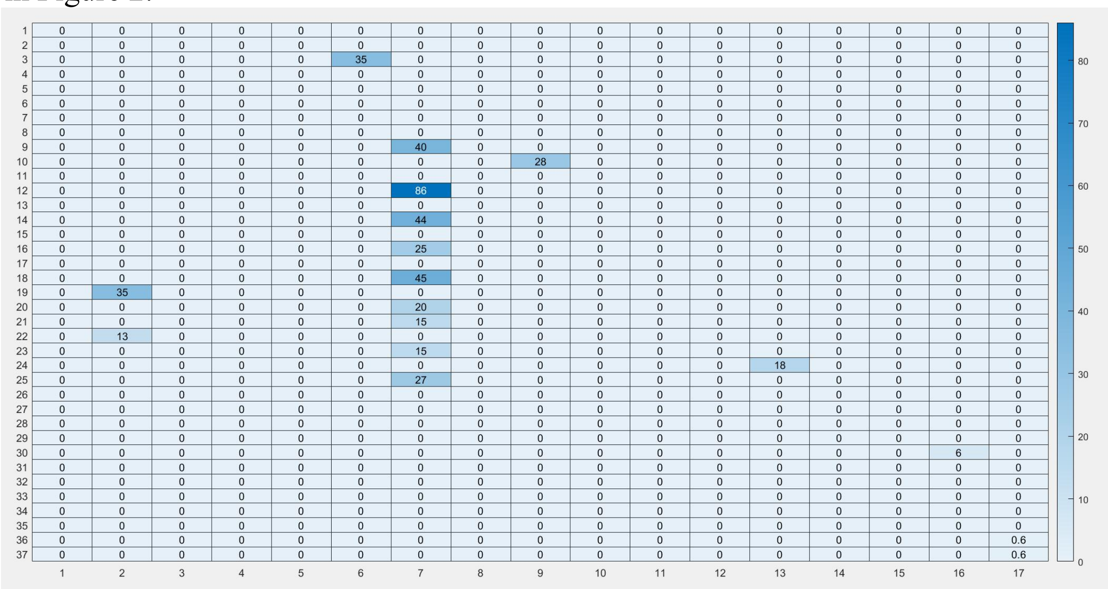

Fig. 2 Heat map of planting trends

4. Conclusion

In this paper, we have presented a novel approach for optimizing crop planting strategies based on Integer Linear Programming (ILP) models and Monte Carlo Simulation (MCS) algorithms. By incorporating data preprocessing, rigorous constraint modeling, and dynamic stochastic planning, we have developed a comprehensive framework that can address the complexities and uncertainties of agricultural planning.

The results of our study demonstrate the effectiveness of the proposed methodology in maximizing farming profits over the specified time horizon. The heatmaps generated from the simulations provide valuable insights into the dynamic trends of planting acreage and profits for different crops, allowing farmers to make informed decisions about their planting strategies.

Overall, our approach represents a significant step forward in the field of agricultural planning and optimization. By leveraging advanced mathematical and simulation techniques, we have demonstrated the potential for significant improvements in profitability and resource management in the agricultural sector. Future research could explore the integration of additional data sources and constraints, as well as the application of more advanced optimization algorithms, to further enhance the robustness and adaptability of the proposed planting strategy.

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